



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

## NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton); Ph. D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from March Number.]

*Corollary II.* But again I am able hence to show that those two straight lines  $AX$ ,  $BX$ , meeting which the straight  $PFHD$  makes either two internal angles toward the same parts equal to two right angles, or consequently (from Eu. I. 13 and 15) alternate external or internal angles equal to one another, or again, from the same cause, an external (as suppose  $DHX$ ) equal to an internal and opposite  $HFX$ ; that, say I, those two straight lines not even in their infinite production can meet one another.

For if from any point  $N$  of  $AX$  is let fall to  $BX$  the perpendicular  $NR$ , this will be in the hypothesis of acute angle (which alone in any case can hinder us) greater (from III. Cor. I.) than the common perpendicular  $KL$ . Therefore those two straight lines  $AX$ ,  $BX$  cannot ever meet one another.

But furthermore here thou hast demonstrated propositions 27 and 28 of the first book of Euclid, and indeed without immediate dependence from the preceding 16 and 17 of the same first book, about which difficulties could arise when the triangle should be of infinite sides on a finite base; to which sort of a triangle without doubt would refer one who believed that these two straight lines  $AX$ ,  $BX$  met one another at least at an infinite distance, although the angles at the transversal  $PFHD$  were such as we have supposed.

Moreover, on account of the demonstrated common perpendicular  $KL$ , surely those two  $KX$ ,  $LX$  cannot come together toward the part of the points  $X$ , since also (from a superposition easily understood) toward the other part also would meet at the same time the remaining and themselves untermiated  $KA$ ,  $LB$ . Wherefore two straight lines  $AX$ ,  $BX$  would enclose a space; which is contrary to the nature of the straight line.

But these things are later. For in the preceding I have never applied either the 16th or 17th of the first book of Euclid, except where clearly it treats of a triangle bounded on every side, as indeed I promised I would so take care to do in *Proemio ad Lectorem*.

[To be Continued.]